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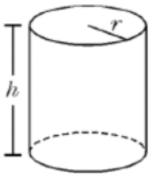
Name:______________________________ Date:____________________

Math 8/9H Section 6.3 Surface Area and Volumes of Cylinders and Cones

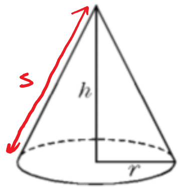
1. Find the surface area and volume of the following shapes:

a) $h = 5cm,\ r = 3cm$



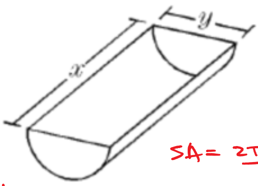
$Vol = \pi r^2 \times h$
$= \pi(9) \times 5$
$= 45\pi\ cm^3$
$SA = 2(\pi r^2)h + 2\pi r(h)$
$= 2(\pi)(9)(5) + 2\pi(3)(5)$
$= 90\pi + 30\pi$
$= 120\pi\ cm^2$

b) $h = 5cm,\ r = 3cm$



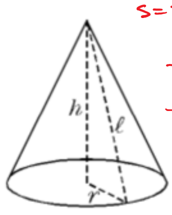
$Vol = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)(5)$
$= 15\pi\ cm^3$
$SA = \pi r^2 + \pi r s$
$= \pi(3)^2 + \pi(3)(5)(\sqrt{34})$
$= 9\pi + 15\sqrt{34}\pi$
$= (9 + 15\sqrt{34})\pi\ cm^2$
$s^2 = 3^2 + 5^2$
$s^2 = 9 + 25$
$s = \sqrt{34}$

c) $x = 12cm,\ y = 6cm$



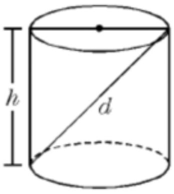
$Vol = \frac{\pi r^2 \times h}{2}$
$= \frac{\pi(3)^2 \times 12}{2}$
$= 54\pi\ cm^3$
$SA = \frac{2\pi r^2 + 2\pi r h}{2} + (x)(y)$
$= \pi(3)^2 + \pi(3)(12) + 6(12)$
$= 9\pi + 36\pi + 72$
$= (45\pi + 72)\ cm$
$h = 12cm$
$r = 3cm.$

d) $h = 36cm,\ l = 39cm$



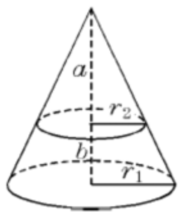
$s = 39cm$
$Vol = \pi r^2 \times h \times \frac{1}{3}$
$= \pi(15)^2 \times 36 \times \frac{1}{3}$
$= 2700\ cm^3$
$SA = \pi r^2 + \pi r s$
$= \pi(15)^2 + \pi(15)(39)$
$= \pi(15)(15) + \pi(15)(39)$
$= \pi(15)(54)$
$= 810\pi\ cm^2$
$r^2 = l^2 - h^2$
$r^2 = 39^2 - 36^2$
$r^2 = (39+36)(39-36)$
$r^2 = 75 \times 3 \rightarrow r = 15cm$

e) $h = 24cm,\ r = 40cm$



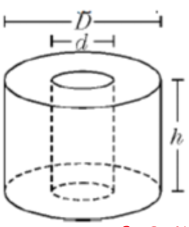
$Vol = \pi r^2 \times h$
$= \pi(40)^2 \times 24$
$= 38400\ cm^3$
$SA = 2\pi r^2 + 2\pi r h$
$= 2\pi(40)(40) + 2\pi(40)(24)$
$= 3200\pi + 1920\pi$
$= 5120\pi\ cm^2.$

f) $r_1 = 15cm,\ r_2 = 10cm,\ b = 5cm,\ a = ?$ What is the volume of the solid with the cone removed?



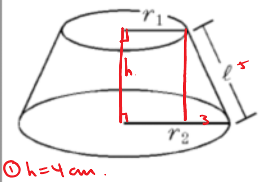
$\frac{a}{10} = \frac{a+5}{15}$
$15a = 10a + 50$
$5a = 50$
$a = 10$
$Vol = V_1 - V_2$
$= \frac{1}{3}(\pi)(15)^2 \times 15 - \frac{1}{3}\pi(10)^2(10)$
$= \frac{1}{3}\pi(15^3 - 10^3)$
$= \frac{1}{3}\pi(2375) = \frac{2375\pi}{3}\ cm^3$

g) $D = 24cm,\ d = 6cm,\ h = 18cm$



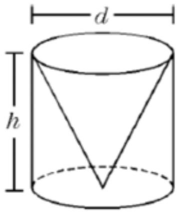
$SA = 2(\pi R^2 - \pi R_2^2) + 2\pi R h$
$SA = 2\pi(12^2 - 3^2) + 2\pi(12)(18)$
$= 270\pi + 432\pi$
$= 702\pi\ cm^2$
$Vol = \pi R^2 h - \pi R_2^2 h$
$= \pi(18)[12^2 - 3^2]$
$= \pi(18)(135)$
$= 2430\pi\ cm^3$

h) $r_1 = 6cm,\ r_2 = 9cm,\ b = 5cm$



$Vol =$
① $h = 4cm.$

2. A cylinder with a height of 10cm and a diameter of 8cm has a cone removed from the inside. What is the volume of the remaining solid?



① A cylinder is 3x the vol of the cone.

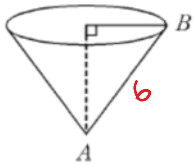
② with the cone removed, the volume remaining is $\frac{2}{3}$ of the cylinder

③ $vol = \frac{2}{3}(\pi) R^2 \times H$

$= \frac{2}{3}(\pi)(4^2)\times 10$

$= \frac{320\pi}{3}\ cm^3$ //.

3. The circumference of the base of the circular cone shown is 10π and AB = 6. Find the height of the cone.



6

$C = 2\pi R = 10\pi$

$2R = 10$

$R = 5cm$

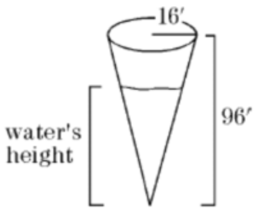
h, 5, 6

$h^2 = 6^2 - 5^2$

$h^2 = 36 - 25$

$h = \sqrt{11}$ //.

4. The water tank in the diagram shown is in the shape of an inverted right circular cone. The radius of its base is 16 feet, and its height is 96 feet. What is the height, in feet, of the water in the tank if the amount of water is 25% of the tank's capacity? Express your answer in simplest radical form.



Method #1: Cones are similar, so use scale factor.

$k^3 = \frac{1}{4}$

$k = \sqrt[3]{\frac{1}{4}} =$ ______

$h = 96 \rightarrow h = 96 \times \sqrt[3]{\frac{1}{4}}$

$h =$ ______

Method 2.

$h = 6R$ b/c $16 \times 6 = 96$.

R, 6R

$\frac{1}{3}\pi R^2(6R) = \frac{1}{3}\pi(16)^2 \times 96 \times \frac{1}{4}$.

$\frac{1}{3}\pi(6)R^3 = \frac{1}{3}\pi(16)^2 \times 96 \times \frac{1}{4}$

$6R^3 = 16 \times 16 \times 96 \times \frac{1}{4}$.

$R^3 = 4 \times 16 \times 16 \times$

$R = \sqrt[3]{4\times4\times4\times4\times4\times4}$

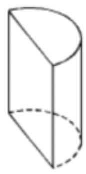
$R = 4 \times \sqrt[3]{4\times4}$

$h = 6 \times 4 \times \sqrt[3]{4\times4}$

$= 24 \times \sqrt[3]{16}$

$=$

5. The solid shown was formed by cutting a circular cylinder in half. If the base has a radius of 6cm and the height is 10cm, what is the total surface area, in terms of π, of the solid?



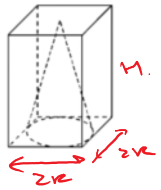
① This solid is half a cylinder + rectangle.

$SA = \frac{2\pi R^2 + 2\pi RH}{2}$ + length × width

$= \pi(6)^2 + \pi(6)(10) + 2(10)$

$= 96\pi + 20$ //.

6. A right circular cone is inscribed in a right prism as shown. What is the ratio of the volume of the cone to the volume of the prism? Express your answer as a common fraction in terms of π

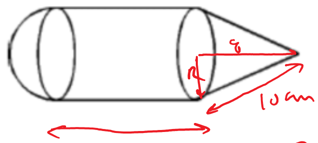


Vol of Prism = 2R × 2R × H.

Vol of Cone = $\frac{1}{3}\pi R^2 \times H$.

Ratio = $\frac{\frac{1}{3}\pi R^2 \times H}{2R \times 2R \times H} = \frac{\frac{1}{3}\pi}{4} = \frac{\pi}{12}$ //

7. A cone with altitude 8cm and slant height 10cm is attached to one end of a cylinder with height 10. A hemisphere with the same circumference as the cylinder is attached to the other end of the cylinder. Find the number of cubic units in the total volume of the solid in terms of π



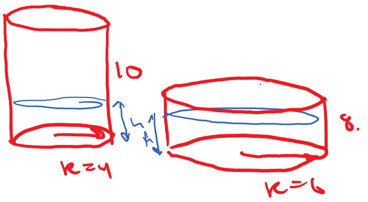
① Vol cone = $\frac{1}{3}\pi R^2 \times H$
= $\frac{1}{3}\pi(36) \times 8$.
= 96π

② cylinder
= $\pi R^2 \times H$.
= $\pi(36) \times 10$
= 360π.

③ hemisphere
= $\frac{1}{2} \times \frac{4}{3}\pi R^3 = \frac{2}{3}\pi(6)^3 = 144\pi$

④ Total = 600π cm³

8. Two cylindrical tanks sit side by side on a level surface. The first tank has a radius of 4meters, a height of 10m, and is full of water. The second tank has a radius of 6m, a height of 8m, and is empty. Water is pumped from the first tank to the second until the depth of water in both tanks is the same. What is the depth of water in each tank (in meters)?



$\pi R^2 \times H = V_1 + V_2$.

$\pi(4)^2 \times 10 = \pi(4)^2 \times H + \pi(6)^2 \times H$

$\pi(160) = \pi(16)H + \pi 36H$.

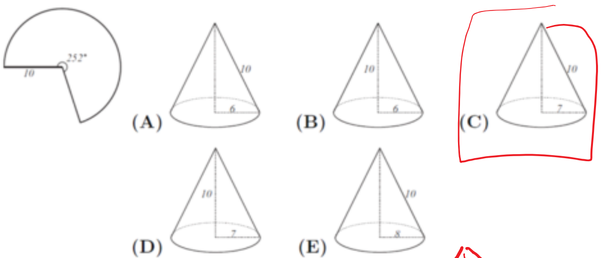
$160 = 52H$.

$\frac{160}{52} = H. = \frac{40}{13}$ cm //

V_1 = water in 1st cylinder

V_2 = water in 2nd cylinder

9. Which of the cones below can be formed from a $252°$ sector of a circle of radius 10 by aligning the two straight sides?



① Circumference of the cone's base

$C = 2\pi R \times \frac{252}{360}$

$= 2 \times \pi \times 10 \times \frac{252}{360}$

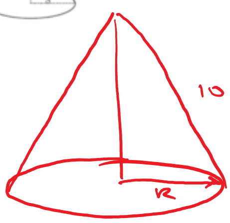
$= \frac{252\pi}{18} = 14\pi$.

$\frac{2 \times 126}{18} = \frac{2 \times 2 \times 63}{2 \times 9}$.

② Radius of the cone.

$2\pi R = 14\pi$

R = 7 //



10. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1cm to a cylinder of radius 3cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28cm, as shown in Figure C. What is the total height, in cm, of the bottle?

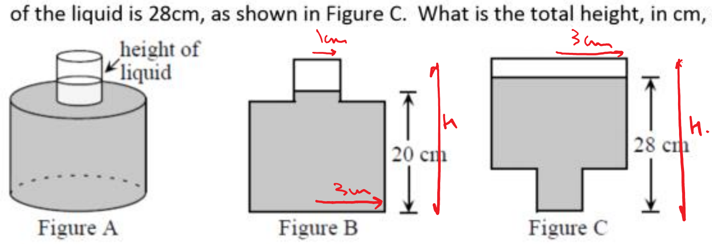


Figure A Figure B Figure C

① USE THE EMPTY SPACE TO CALCULATE THE Vol.

$\pi R_1^2 \times H_1 = \pi R_2^2 \times H_2$

$\pi(1cm)^2 \times (H-20) = \pi(3)^2 \times (H-28)$

$H - 20 = 9H - 9(28)$

$9(28) - 20 = 8H$

$232 = 8H$

$29 = H$

11. A student has two open-topped cylindrical containers. (the walls of the two containers are thin enough so that their width can be ignored) The larger container has a height of 20cm, a radius of 6cm and contains water to a depth of 17cm. The smaller container has a height of 18cm, a radius of 5cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to:

A) 2.82 cm B) 2.84 cm C) 2.86cm D) 2.88cm E) 2.90

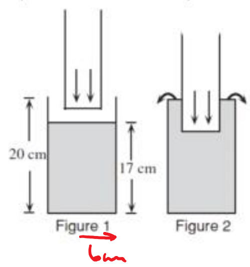
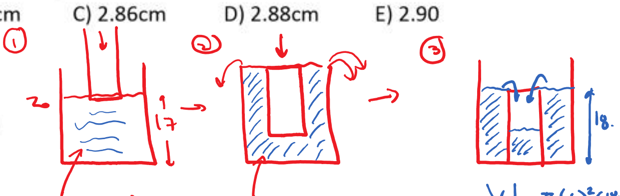


Figure 1 Figure 2



① $Vol = \pi(6)^2 \times 17$

② $Vol = \pi(6)^2(20) - \pi(5)^2(18)$

$= \pi(720) - \pi(450)$

$= \pi(270)$

THIS AMOUNT OF WATER WILL STAY IN

③ HEIGHT OF WATER IN SMALL CYLINDER

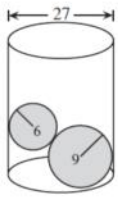
$Vol = \pi(6)^2(18) - \pi(5)^2(18) + \pi(5)^2(H) = 270\pi$

$= \pi(18)(11) + \pi(25)(H) = 270\pi$

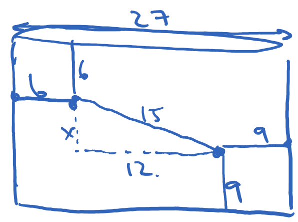
$198 + 25H = 270$

$H = \frac{72}{25}$

12. Challenge: A cylinder, which has a diameter of 27 and a height of 30, contains two lead spheres with radii 6 and 9, with the larger sphere sitting on the bottom of the cylinder. Water is poured into the cylinder so that it just covers both spheres. What is the volume of the water required?



① USE THE CENTERS



$x^2 + 12^2 = 15^2$

$x^2 = 81$

$x = 9$

225
144
81

② Total HEIGHT OF WATER

$H = 6 + 9 + 9$

$= 24\ cm$

$R = \frac{27}{2}$

③ $Vol\ water = \pi R^2 \times H - \frac{4}{3}\pi R_1^3 - \frac{4}{3}\pi R_2^3$

$= \pi\left(\frac{27}{2}\right)^2(24) - \frac{4}{3}\pi(6\times6\times6) - \frac{4}{3}\pi(9\times9\times9)$

$= \pi(27)^2(6) - \pi(4\times6\times6\times2) - \pi(4\times9\times9\times3)$

$= 4374\pi - 288\pi - 972\pi$

$= 3114\pi$